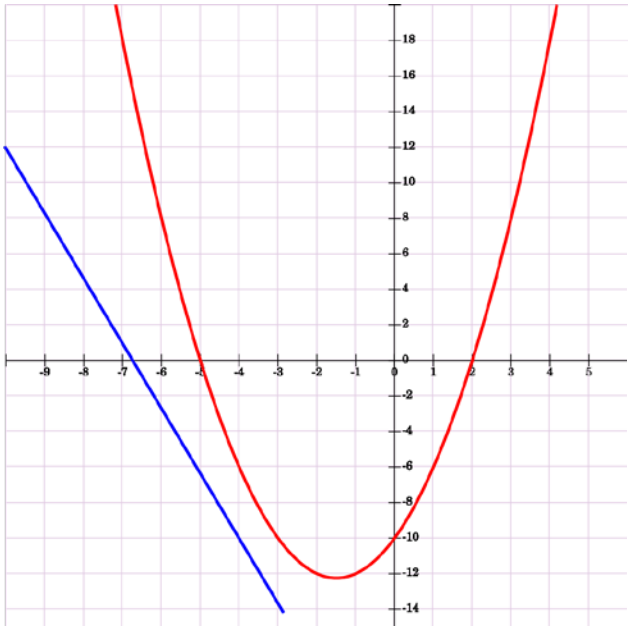
Math Review – Part C

Advanced Level

(Up to end of MAT 053)



A scientific calculator is allowed.

Answers provided in the final section.

Advanced Level Algebra

ALGEBRAIC EXPRESSIONS

Simplify the following algebraic expressions by combining like terms.

1. $2xy^2 - 5xy - 3xy^2 + 2xy$

2. $5x - 7(2x - 3) - 4$

3. $\frac{1}{2}(6x + 4) - \frac{2}{3}(3x - 9)$

A summary of the **Laws of Exponents** is given below.

$x^1 = x$ $\quad$ $x^0 = 1\,(\text{where } x \neq 0)$ $\quad$ $x^{-n} = \frac{1}{x^n}$

$x^m \bullet x^n = x^{m+n}$ $\quad$ $\frac{x^m}{x^n} = x^{m-n}$ $\quad$ $(x^m)^n = x^{mn}$

Using the above laws, complete the following simplifications.

4. $(3x^2)(5x)(x^2)^3$

5. $\frac{42x^6y^7}{-21x^{10}y^{-3}}$

6. $\left(\frac{4a^{-3}b^2}{-8ab^3}\right)^3$

LINEAR EQUATIONS AND INEQUALITIES

Solve each of the following linear equations.

7. $5x + 7 = 2x - 8$

8. $25 - (x + 1) = 3x$

9. $\frac{3x}{2} + 5 = \frac{x}{3}$

10. $0.2x + 4.9 = 0.7x - 1.3$

Solve each of the following inequalities.

11. $2x \leq x - 8$

12. $3 - 5x \geq x + 12$

13. $-4 < 2x + 2 \leq 8$

Write an algebraic equation or inequality to represent each problem. Then solve the problem using your equation or inequality. Be sure to identify the variable.

14. The sum of two consecutive odd integers is 56. Find the integers.
    Let the 1st number = x
    Let the 2nd number = $x+2$

15. Ms. Jones earns a base salary of $880 per month, plus $32 for each product that she sells. During November she earned a total of $5680. How many products did she sell?

16. Tickets for a local concert sold for $5 and $8. Jim collected $499 on the sale of 80 tickets. How many of each type did he sell?

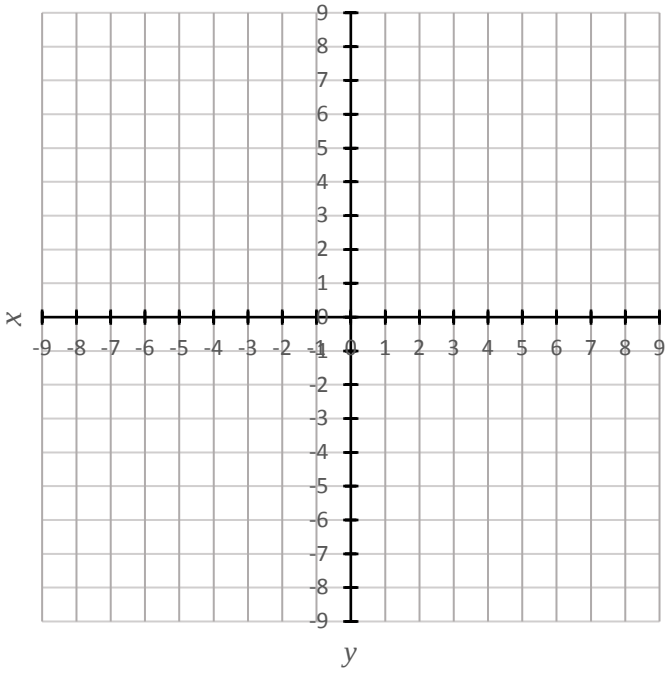
17. You have $20,000 to invest. You invest part of this at 6% interest and the rest at 8%. What is the most you can invest at 6% to earn at least $1440 interest?

GRAPHING OF LINEAR EQUATIONS AND INEQUALITIES

Graph each of the following.

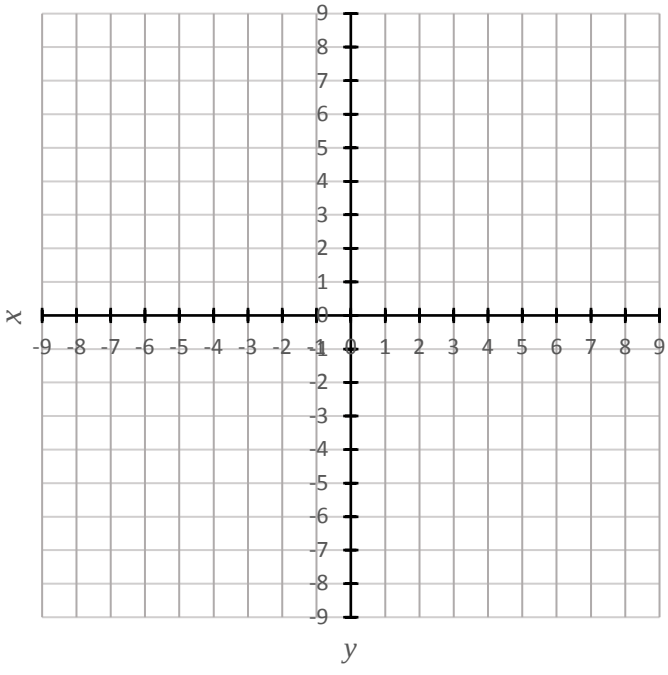
18. $y = \frac{1}{2}x - 4$

x	y



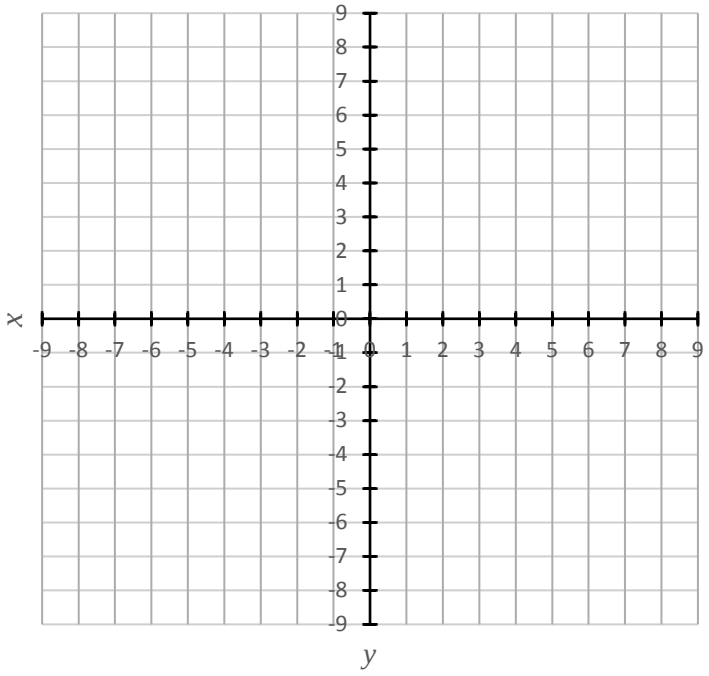
19. $3x + 2y = 6$

x	y



20. $2y > 6x + 4$

x	y



SYSTEMS OF EQUATIONS

A solution of a system of equations in two variables is an ordered pair of numbers that makes both equations true.

Solve the following systems of equations.

21. $x + y = 4$
$x - y = 2$

22. $y = 7 - x$
$2x - y = 8$

23. $5x + 3y = 17$
$-5x + 2y = 3$

FACTORING POLYNOMIALS

Factor each of the following.

24. $x^2 + 10x + 25$

25. $3x^4 - 12$

26. $15x^2 + 7x - 2$

27. $8x^3 + 125$

RATIONAL EXPRESSIONS AND EQUATIONS

Simplify.

28. $\frac{x^4}{3x+6} \bullet \frac{5x+10}{5x^2}$

29. $\frac{y^2-9}{y+2} \div \frac{y+3}{y+2}$

30. $\frac{3x}{7} + \frac{2y}{3x}$

31. $\frac{a}{a+3} - \frac{a-4}{a}$

Solve each equation.

32. $\frac{4}{5} + \frac{x}{3} = \frac{x}{5}$

33. $\frac{3}{y+1} = \frac{2}{y-3}$

34. $\frac{50}{x} - \frac{50}{x-2} = \frac{4}{x}$

Solve each problem using an algebraic equation.

35. A tank can be filled in 18 hours by pipe A alone and in 22 hours by pip B alone. How long will it take to fill the tank if both pipes are in operation?

36. The speed of a river is 5 km per hour. A boat travels 7 km upstream in the same time as it takes to travel 16 km downstream. What would be the speed of the boat in still water?

RADICAL EXPRESSIONS AND EQUATIONS

Simplify by combining like radical terms.

37. $2\sqrt{3}-5\sqrt{5}+4\sqrt{3}-2\sqrt{5}$

38. $3\sqrt{45}+3\sqrt{5}$

39. $\sqrt{25x-25}-\sqrt{9x-9}$

Multiply and simplify the following.

40. $\sqrt{5}(2+3\sqrt{5})$

41. $(2-\sqrt{3})(2+\sqrt{3})$

42. $(\sqrt{5}+\sqrt{2})(2\sqrt{5}-3\sqrt{2})$

Simplify.

43. $\sqrt{16x^4}$

44. $\sqrt[3]{-8}$

45. $\sqrt[8]{(-2)^8}$

Solve each equation.

46. $\sqrt{x}-7=3$

47. $x+1=3\sqrt{x-1}$

48. $\sqrt{5x-3}=\sqrt{2x+3}$

QUADRATIC EQUATIONS

The quadratic formula below is important; be sure to remember it.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve each of the following quadratic equations.

49. $x^2 - 6x + 9 = 0$

50. $x^2 - 2x - 5 = 0$

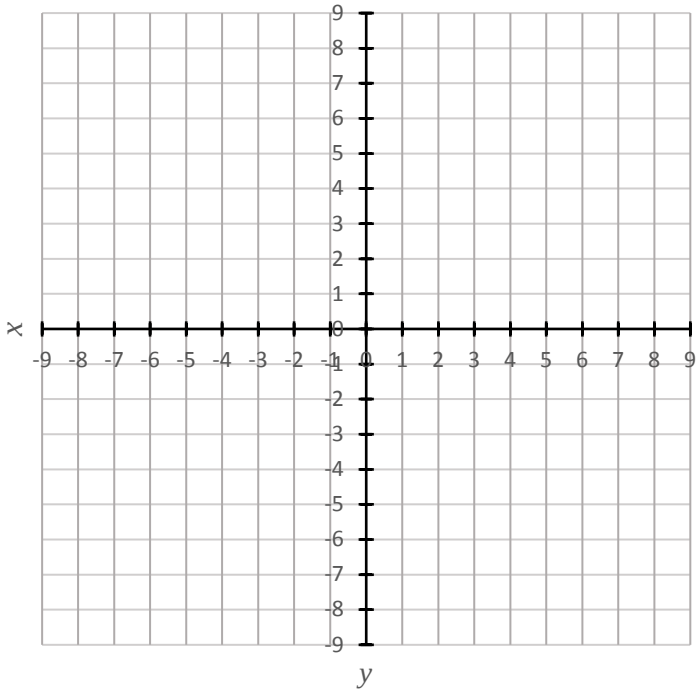
51. $4x^2 + 5x - 6 = 0$

52. $\frac{x+3}{x} = \frac{x-4}{3}$

Graph each of the following questions.

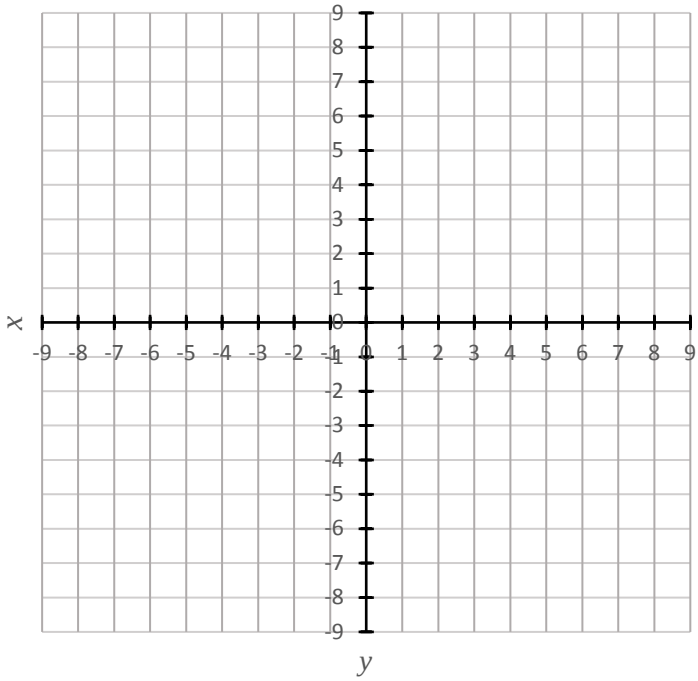
53. $y = (x+2)^2 - 3$

x	y



54. $y = -2(x-1)^2 + 1$

x	y



FUNCTIONS

Given $f(x) = x^2$ and $g(x) = 3x + 2$, find the following.

55. $f(-2)$

56. $g\left(\frac{1}{3}\right)$

57. $(f + g)(x)$

Advanced Level Trigonometry

TRIGONOMETRIC RATIOS

The **trigonometric ratios** are:

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

The **Law of Sines** is:

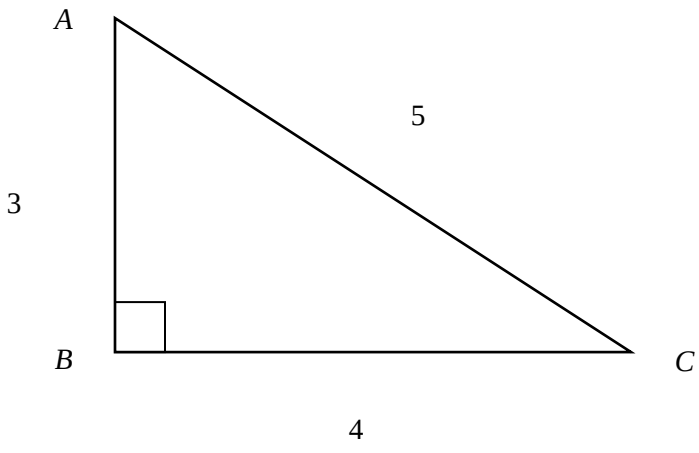
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The **Law of Cosines** is:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

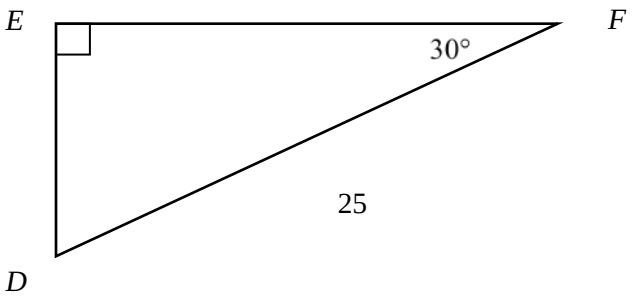
Use the above information or a scientific calculator to answer the following questions.

58. Find the measure of $\angle ACB$

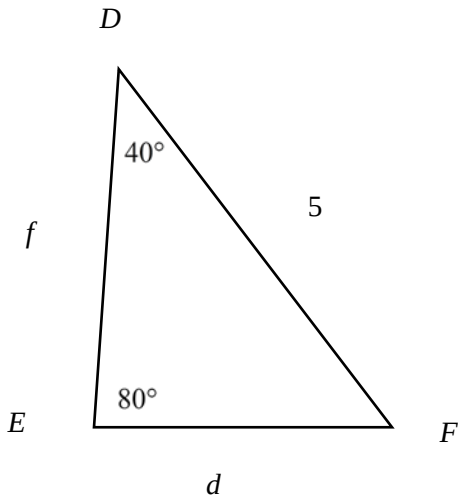


59. Find the length of $\overline{EF}$

$\overline{EF}$ =



60. Solve the ΔDEF by finding the measure of each of those sides and angles whose quanitty is not given and is represented only by letter.

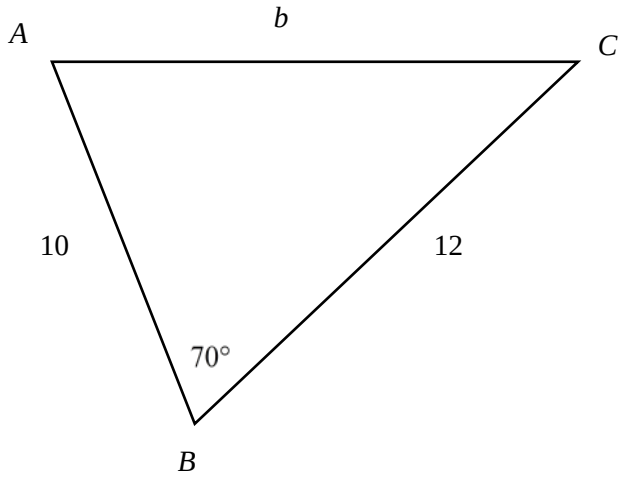


$\angle F$ =

f =

d =

61. Solve the ΔABC by finding the measure of each of those sides and angles represented only by letter.



$b =$

$\angle A =$

$\angle C =$

Answers – Part C

Algebraic Expressions

1. $-xy^2 - 3xy$
2. $17 - 9x$
3. $x + 8$
4. $15x^9$
5. $-2x^{-4}y^{10}$ *or* $\frac{-2y^{10}}{x^4}$
6. $-\frac{1}{8a^{12}b^3}$

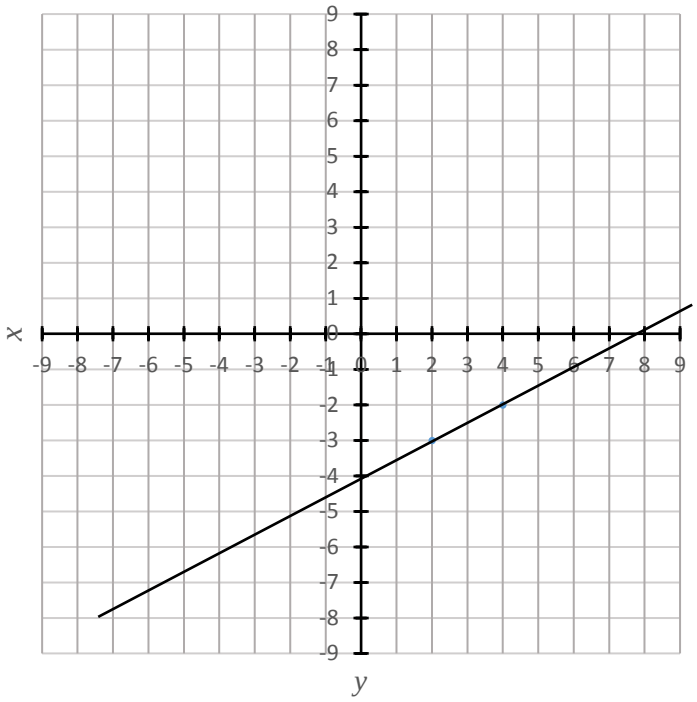
Linear Equations and Inequalities

7. $x = -5$
8. $x = 6$
9. $x = -\frac{30}{7}$
10. $x = 12.4$
11. $x \le -8$
12. $x \le \frac{-3}{2}$
13. $-3 < x \le 3$
14. $n + n + 2 = 56$ The numbers are 27 and 29
15. $880 + 32n = 5680$ Number of products sold is 150
16. $5n + 8(80 - n) = 499$ 47 tickets at $5 and 33 tickets at $8
17. $6\%n + 8\%(20000 - n) \ge 1440$ $8,000 max.

Graphing of Linear Equations and Inequalities

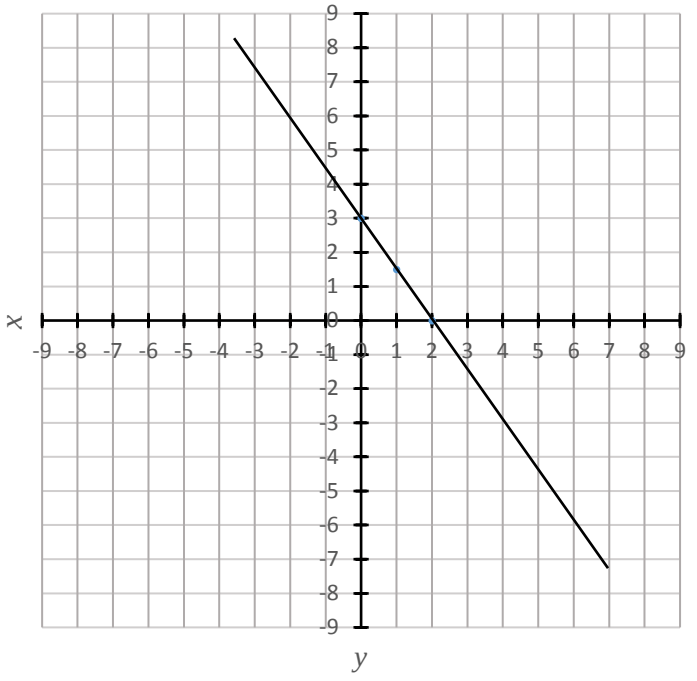
18. $y = \frac{1}{2}x - 4$

x	y
2	-3
4	-2



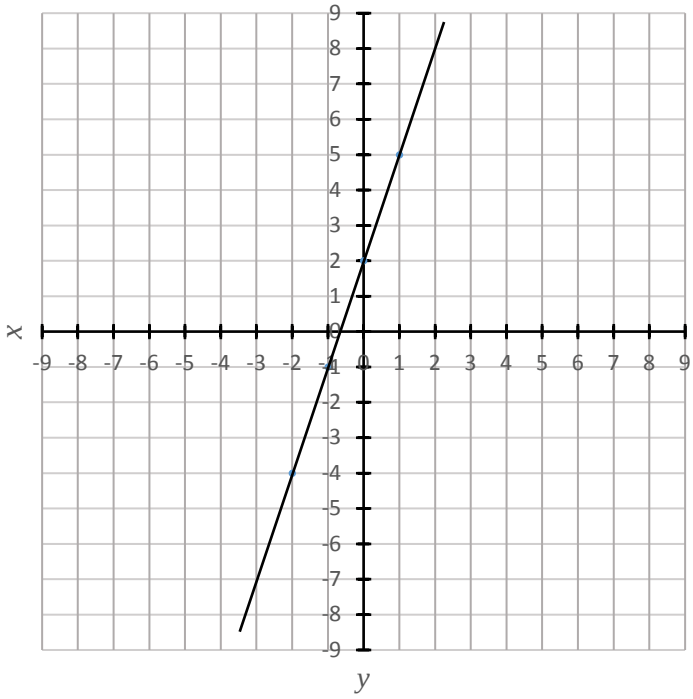
19. $3x + 2y = 6$

x	y
2	0
1	1.5
0	3



20. $2y > 6x + 4$

x	y
1	5
0	2
-1	-1
-2	-4



System of Equations

21. $(3,1)$

22. $(5,2)$

23. $(1,4)$

Factoring Polynomials

24. $(x+5)(x+5)$ *or* $(x+5)^2$

25. $3(x^4-4) = 3(x^2+2)(x^2-2)$

26. $(3x+2)(5x-1)$

27. $(2x+5)(4x^2-10x+25)$

Rational Expressions and Equations

28. $\frac{x^2}{3}$

29. $y-3$

30. $\frac{9x^2+14y}{21x}$

31. $\frac{a+12}{a(a+3)}$

32. $x=-6$

33. $y=11$

34. $x=-23$

35. $\frac{1}{18}+\frac{1}{22}=\frac{1}{t}$ time is $9\frac{9}{10}$ or 9 hours and 54 minutes.

36. $\frac{7}{r-5}=\frac{16}{r+5}$ speed is 12.8 km per hour

Radical Expressions and Equations

37. $6\sqrt{3}-7\sqrt{5}$

38. $12\sqrt{5}$

39. $2\sqrt{x-1}$

40. $2\sqrt{5}+15$

41. $4-3=1$

42. $4-\sqrt{10}$

43. $4x^2$

44. -2

45. 2

46. $x=100$

47. $x=2 \text{ or } 5$

48. $x=2$

Quadratic Equations

49. $x=3$

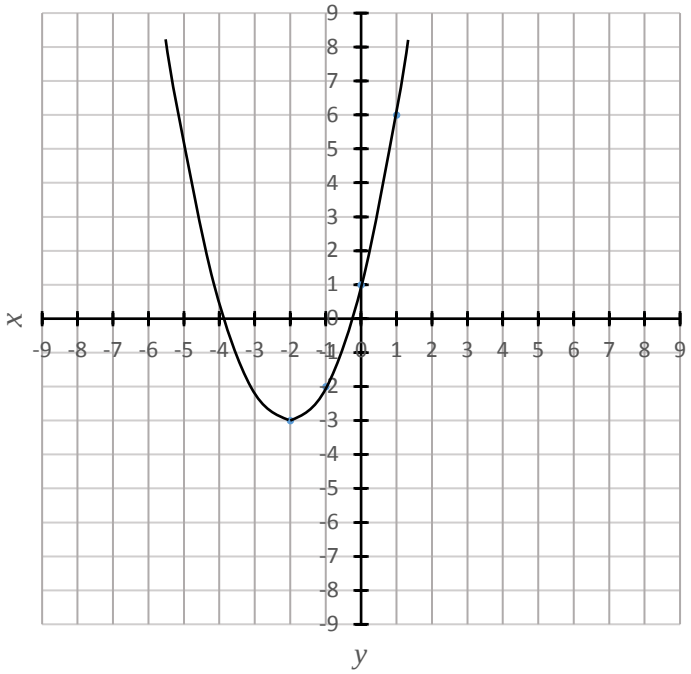
50. $x=1\pm\sqrt{6}$ *or* 3.45 and -1.45

51. $x=\frac{3}{4}$ *and* -2

52. $x=\frac{7\pm\sqrt{85}}{2}$ *or* 8.1 and -1.1

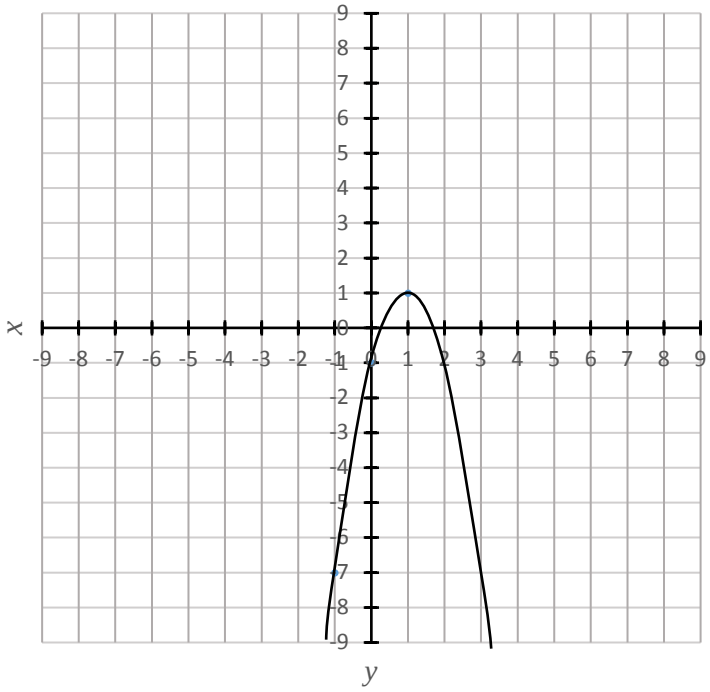
53. $y = (x+2)^2 - 3$

x	y
0	1
1	6
-1	-2
-2	-3



54. $y = -2(x-1)^2 + 1$

x	y
0	-1
1	1
-1	-7



Functions

55. 4

56. 3

57. $x^2 + 3x + 2$

Trigonometric Ratios

58. the measure of $\angle BCA = 36.9°$

59. the measure of $\overline{EF}$ = 21.7 (units)

60. $\angle F = 180° - (80° + 40°) = 60°$

$$\frac{d}{\sin 40°} = \frac{5}{\sin 80°} \quad d = 3.3$$

$$\frac{f}{\sin 60} = \frac{5}{\sin 80} \quad f = 4.4$$

61. $b^2 = 12^2 + 10^2 - 2 \times 12 \times 10 \times \cos 70°$

$b = 12.725$

$$\frac{12}{\sin A} = \frac{12.725}{\sin 70°}$$

$\angle A = 62.4°$

$\angle C = 180° - (70° + 62.4°) = 47.6°$

Table of Trigonometric Ratios

Angle	tan	sin	cos	Angle	tan	sin	cos
1°	0.017	0.017	1.000	**46°**	1.036	0.719	0.695
2°	0.035	0.035	0.999	**47°**	1.072	0.731	0.682
3°	0.052	0.052	0.999	**48°**	1.111	0.743	0.669
4°	0.070	0.070	0.998	**49°**	1.150	0.755	0.656
5°	0.087	0.087	0.996	**50°**	1.192	0.766	0.643
6°	0.105	0.105	0.995	**51°**	1.235	0.777	0.629
7°	0.123	0.122	0.993	**52°**	1.280	0.788	0.616
8°	0.141	0.139	0.990	**53°**	1.327	0.799	0.602
9°	0.158	0.156	0.988	**54°**	1.376	0.809	0.588
10°	0.176	0.174	0.985	**55°**	1.428	0.819	0.574
11°	0.194	0.191	0.982	**56°**	1.483	0.829	0.559
12°	0.213	0.208	0.978	**57°**	1.540	0.839	0.545
13°	0.231	0.225	0.974	**58°**	1.600	0.848	0.530
14°	0.249	0.242	0.970	**59°**	1.664	0.857	0.515
15°	0.268	0.259	0.966	**60°**	1.732	0.866	0.500
16°	0.287	0.276	0.961	**61°**	1.804	0.875	0.485
17°	0.306	0.292	0.956	**62°**	1.881	0.883	0.469
18°	0.325	0.309	0.951	**63°**	1.963	0.891	0.454
19°	0.344	0.326	0.946	**64°**	2.050	0.899	0.438
20°	0.364	0.342	0.940	**65°**	2.145	0.906	0.423
21°	0.384	0.358	0.934	**66°**	2.246	0.914	0.407
22°	0.404	0.375	0.927	**67°**	2.356	0.921	0.391
23°	0.424	0.391	0.921	**68°**	2.475	0.927	0.375
24°	0.445	0.407	0.914	**69°**	2.605	0.934	0.358
25°	0.466	0.423	0.906	**70°**	2.748	0.940	0.342
26°	0.488	0.438	0.899	**71°**	2.904	0.946	0.326
27°	0.510	0.454	0.891	**72°**	3.078	0.951	0.309
28°	0.532	0.469	0.883	**73°**	3.271	0.956	0.292
29°	0.554	0.485	0.875	**74°**	3.487	0.961	0.276
30°	0.577	0.500	0.866	**75°**	3.732	0.966	0.259
31°	0.601	0.515	0.857	**76°**	4.011	0.970	0.242
32°	0.625	0.530	0.848	**77°**	4.332	0.974	0.225
33°	0.649	0.545	0.839	**78°**	4.705	0.978	0.208
34°	0.675	0.559	0.829	**79°**	5.145	0.982	0.191
35°	0.700	0.574	0.819	**80°**	5.671	0.985	0.174
36°	0.727	0.588	0.809	**81°**	6.314	0.988	0.156
37°	0.754	0.602	0.799	**82°**	7.115	0.990	0.139
38°	0.781	0.616	0.788	**83°**	8.144	0.993	0.122
39°	0.810	0.629	0.777	**84°**	9.514	0.995	0.105
40°	0.839	0.643	0.766	**85°**	11.430	0.996	0.087
41°	0.869	0.656	0.755	**86°**	14.301	0.998	0.070
42°	0.900	0.669	0.743	**87°**	19.081	0.999	0.052
43°	0.933	0.682	0.731	**88°**	28.636	0.999	0.035
44°	0.966	0.695	0.719	**89°**	57.290	1.000	0.017
45°	1.000	0.707	0.707				

Table of Squares & Square Roots

n	n^2	n	n	n^2	n
1	1	1.000	**51**	2601	7.141
2	4	1.414	**52**	2704	7.211
3	9	1.732	**53**	2809	7.280
4	16	2.000	**54**	2916	7.348
5	25	2.236	**55**	3025	7.416
6	36	2.449	**56**	3136	7.483
7	49	2.646	**57**	3249	7.550
8	64	2.828	**58**	3364	7.616
9	81	3.000	**59**	3481	7.681
10	100	3.162	**60**	3600	7.746
11	121	3.317	**61**	3721	7.810
12	144	3.464	**62**	3844	7.874
13	169	3.606	**63**	3969	7.937
14	196	3.742	**64**	4096	8.000
15	225	3.873	**65**	4225	8.062
16	256	4.000	**66**	4356	8.124
17	289	4.123	**67**	4489	8.185
18	324	4.243	**68**	4624	8.246
19	361	4.359	**69**	4761	8.307
20	400	4.472	**70**	4900	8.367
21	441	4.583	**71**	5041	8.426
22	484	4.690	**72**	5184	8.485
23	529	4.796	**73**	5329	8.544
24	576	4.899	**74**	5476	8.602
25	625	5.000	**75**	5625	8.660
26	676	5.099	**76**	5776	8.718
27	729	5.196	**77**	5929	8.775
28	784	5.292	**78**	6084	8.832
29	841	5.385	**79**	6241	8.888
30	900	5.477	**80**	6400	8.944
31	961	5.568	**81**	6561	9.000
32	1024	5.657	**82**	6724	9.055
33	1089	5.745	**83**	6889	9.110
34	1156	5.831	**84**	7056	9.165
35	1225	5.916	**85**	7225	9.220
36	1296	6.000	**86**	7396	9.274
37	1369	6.083	**87**	7569	9.327
38	1444	6.164	**88**	7744	9.381
39	1521	6.245	**89**	7921	9.434
40	1600	6.325	**90**	8100	9.487
41	1681	6.403	**91**	8281	9.539
42	1764	6.481	**92**	8464	9.592
43	1849	6.557	**93**	8649	9.644
44	1936	6.633	**94**	8836	9.695
45	2025	6.708	**95**	9025	9.747
46	2116	6.782	**96**	9216	9.798
47	2209	6.856	**97**	9409	9.849
48	2304	6.928	**98**	9604	9.899
49	2401	7.000	**99**	9801	9.950
50	2500	7.071	**100**	10000	10.000